COMPUTER-AIDED ENGINEERING
Ph.D. QUALIFIER EXAM – FALL 2015

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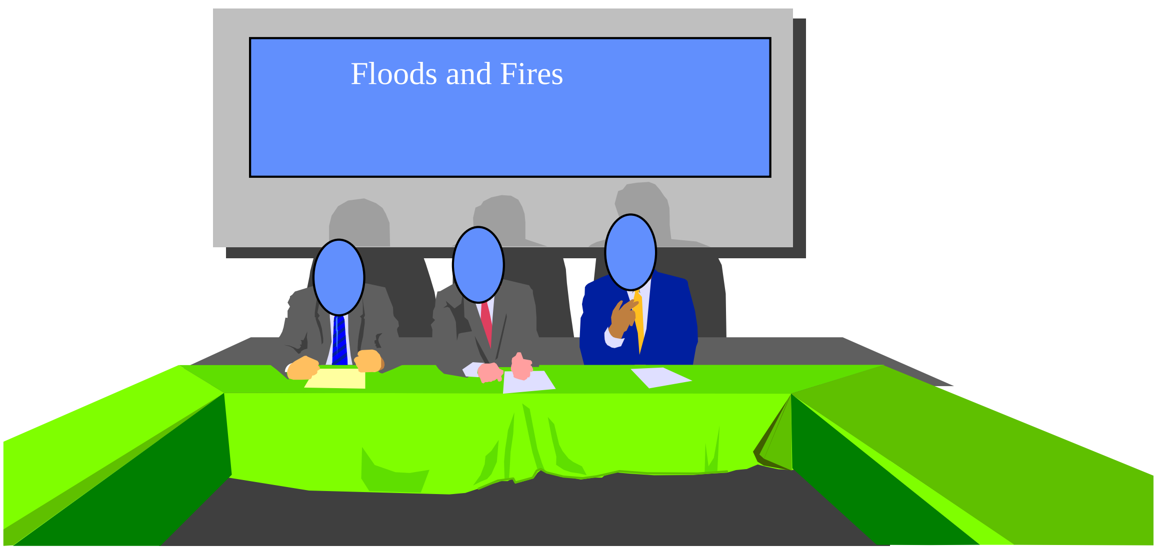


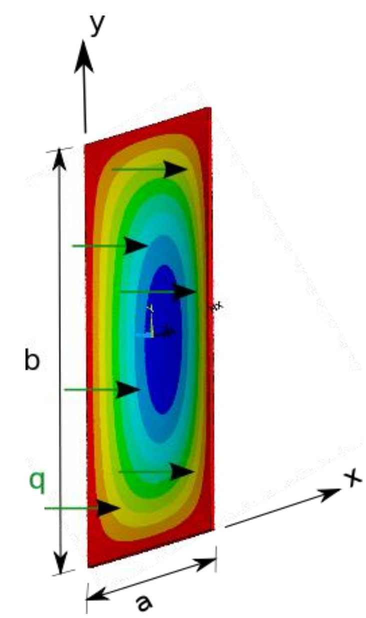
- All questions in this exam have a common theme: ***Floods and Fires***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

During SuperStorm Sandy in 2012, some of the subways in New York City flooded. In preparation for future super storms, a company is developing plastic sheets that can be deployed to cover subway staircase openings (this is true). To perform an initial engineering assessment of these plastic sheets, it is of interest to determine their maximum deflections and deflected shapes.

The plastic sheets will be modeled as rectangular plates, the deflected shape of a sample is shown to the right. The deflection equation for a simply supported rectangular plate subject to a uniform load is



$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{16q_0}{(2m-1)(2n-1)\pi^6 D}\left[\frac{(2m-1)^2}{a^2}+\frac{(2n-1)^2}{b^2}\right]^{-2} \times \sin\frac{(2m-1)\pi x}{a}\sin\frac{(2n-1)\pi y}{b}.$$

where D is the flexural rigidity: $D = \dfrac{Et^3}{12(1-\nu^2)}$

For this problem, let's just consider a horizontal cross-section of the plate through its center, that is, we will set $y = b/2$. Assume that the distributed load q is directed in the positive Z direction. Further assume that plate length $a = 4$ m and plate height $b = 3$ m.

Answer the following questions:

a) Assume the deflected plate cross-section model is a cubic Bezier curve. Sketch the plate shape assuming that q > 0 and plot the control vertices. Indicate where the maximum plate deflection occurs.

b) Now, assume the deflected plate cross-section model is a composite cubic Bezier curve (consisting of two curves), where the model acknowledges the symmetry of the plate deflection. Sketch the plate shape assuming that $q > 0$ and plot the control vertices. Specify the continuity condition achieved and explain how this condition is achieved. Indicate where the maximum plate deflection occurs.

c) Given that the actual deflected plate follows an infinite sum of sinusoidal curves (given in the $w(x,y)$ equation above), what do you think is an appropriate type of geometric model for the plate? Specify surface equation type, its degree, and justify.

d) Now, realize that in a real situation, the plastic sheet will not be subject to a uniform load, due to the non-uniform water loading (greater pressure near the bottom of the plate). The rectangular plate solution will still be an infinite sum of sinusoidal terms, but their amplitudes will change. How would your answer to c) change given this more realistic loading condition?

General Bezier curve modeling. For a cubic Bezier curve with the following control vertices:

$\mathbf{p}_0 = (0, 0)$, $\mathbf{p}_1 = (1, 3)$, $\mathbf{p}_2 = (3, 4)$, $\mathbf{p}_3 = (6, 0)$,

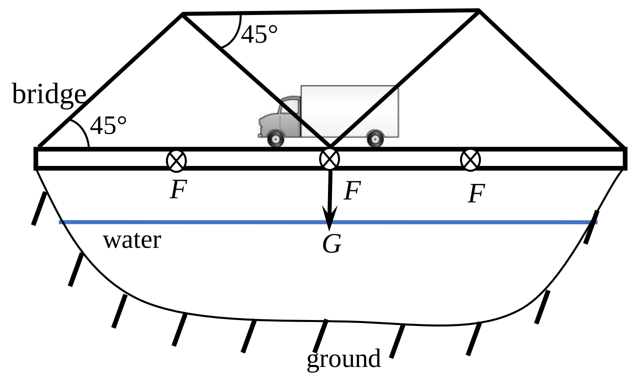
e) Derive the equations for the curve. Simplify the equations into the form:

$$\mathbf{a}_3 u^3 + \mathbf{a}_2 u^2 + \mathbf{a}_1 u + \mathbf{a}_0 = \mathbf{p}(u)$$

f) Compute the point on this curve at $u = 0.3$.

Question 2 – Finite-Element Analysis

As an engineer, you are assigned a task to quickly verify the integrity of a bridge under an emergency weather condition, as illustrated on the right.



The truck passing through the bridge adds an external load (***G***) in the middle of the bridge. The pressure from wind is simplified as three evenly distributed loads (***F***'s) applied perpendicularly (to the plane of the paper) at the main bridge.

(a) Build a finite-element model to estimate the deformation where the load ***G*** is applied when the wind load ***F***'s are NOT considered. Make necessary assumptions of lengths, cross-section areas, materials, etc.

1. Clearly state all of your simplifications and assumptions used in your model.
2. Specifically show the boundary conditions and loading conditions for each one of the nodes you use separately.
3. Write down the element stiffness matrix and assembly stiffness matrix.

(b) Discuss briefly how the model would be different if the wind load ***F***'s are considered in the structural analysis?

<u>Element A - Stiffness Matrix</u>

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; $l=(x_2-x_1)/L$ and $m=(y_2-y_1)/L$ are directional *cos*() and *sin*() respectively.

<u>Element B - Stiffness Matrix</u>

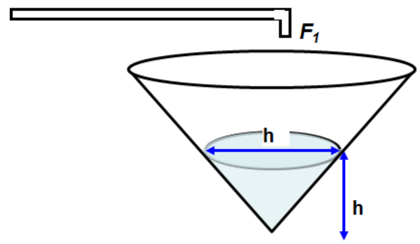
$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$

where E, I, and L are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Analysis

South Carolina decided to construct anti-flood water reservoirs to lower water levels and mitigate the dangers from crack propagations in the dams.

You are designing a cone shaped tank to hold water for a village in South Carolina. The volume of liquid it holds, can be computed as $V=\pi h^3/12$ m^3 as shown in the figure.



a) What depth must the tank be filled to, so that it holds 20π m^3? Apply the Newton-Raphson method with the initial guess as the exact solution (use two significant figures as an initial guess). Conduct three iterations and present the results clearly.
b) How can you assure that your answer is close enough to the exact solution without knowing the solution?

When it rains, the tank is filling with water at a constant rate of $F_1(t) = \boldsymbol{F} = \pi$ m^3/min. Also, water evaporates from the tank at a rate proportional to the surface area of liquid. The constant of proportionality is $\alpha = 0.01$ m/min. The initial level of h_0 is 10 m.

c) Write an equation for the continuous-time model of the tank level, h(t). With a step size of 1 min, use the RK-4 method to find h(1).
d) When a step size of 0.25 is used, by how much would the truncation error in h(1) decrease? Answer this question without solving the problem.
e) Explain the mathematical or graphical meaning of the k's in the RK-4 methods (see equations below).

Note: The equations for RK4 method are:

$$x_{n+1} = x_n + (h/6)(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4})$$

$$k_{n1} = f(t_n, x_n),$$

$$k_{n2} = f(t_n + (h/2),\ x_n + (h/2)k_{n1})$$

$$k_{n3} = f(t_n + (h/2),\ x_n + (h/2)k_{n2})$$

$$k_{n4} = f(t_n + h,\ x_n + hk_{n3})$$