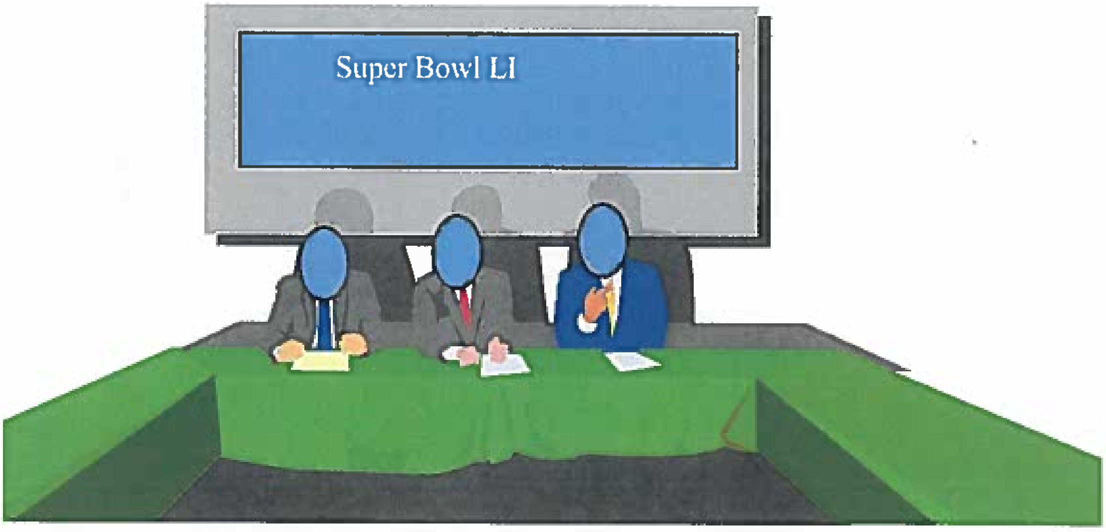
COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – SPRING 2017

THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332-0405

S.-K. Choi, C. Paredis, S. Sitaraman (Chair), and Y. Wang



- All questions in this exam have a common theme: ***Super Bowl LI***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

Suppose that you have a task to design a mold to build Super Bowl rings.

You want to use a bi-cubic Bézier surface patch to model a portion of the ring.

a) Derive the equation of the bi-quadratic Bézier surface patch in a matrix form.

b) You received the design of one patch of the ring outer surface with the control points as

Patch 1:

(1 17 2)	(3 17 6)	(6 17 5)
(1 14 2)	(3 14 6)	(6 14 5)
(1 11 2)	(3 11 3)	(6 11 4)

To start the design of the second patch, we have decided some of the control points as follows.

Patch 2:

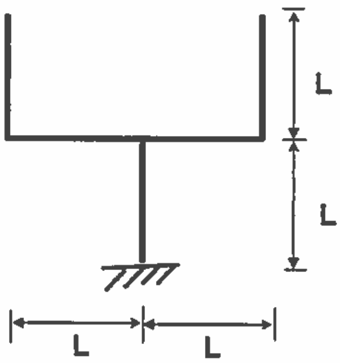
(-5 17 -5)	**P_4**	**P_1**
(-5 14 -5)	**P_5**	**P_2**
(-5 11 -4)	**P_6**	(1 11 2)

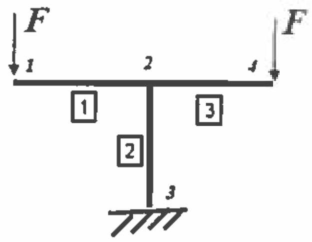
To ensure the C^0 and C^1 continuity between the two patches, what are the coordinate values of **P_1**, **P_2**, **P_3**, **P_4**, **P_5**, and **P_6**? Explain how you decide the values in detail and show calculations.

c) Calculate the unit normal vector of Patch 1 at point (1 17 2).

Question 2 – Finite-Element Analysis

The figure below shows a schematic of a football goal post. In order to ensure that the horizontal section remains horizontal, you are asked to determine the tip dispalcement at the node 1. You can assume that the weights of the vertical posts are considered to be applied loads as illustrated in the figure with F. Also, both the vertical and horizontal posts have a length of L and are cylindrical with a diameter of D. To faciliate the analysis procedure of the post, you may assume that the cylindrical members can be modeled with the **Element B,** which is modified from the traditional **Element A** by ignoring the rotational effects.





1. Use finite-element formulation to solve for the tip displacement by using **Element B.**
2. *Minimize* the number of elements in your FEA model and show the boundary conditions and loading conditions clearly.
3. Starting with the element matrices, write down the assembly stiffness matrix.
4. Show all steps to determine the tip displacement at node 1.
5. What is the expected result when you use **Element A** rather than **Element B**?

Element A - Stiffness Matrix

$$[K]=\begin{bmatrix} AE/L & 0 & 0 & -AE/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ -AE/L & 0 & 0 & AE/L & 0 & 0 \\ 0 & -12EI/L^3 & 6EI/L^2 & 0 & 12EI/L^3 & 6EI/L^2 \\ 0 & -6EI/L^2 & 2EI/L & 0 & 6EI/L^2 & 4EI/L \end{bmatrix} \begin{matrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{matrix}$$

where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively.

Element B - Stiffness Matrix

$$[K]=\begin{bmatrix} m_1 & m_2 & -m_1 & -m_2 \\ m_2 & m_3 & -m_2 & -m_3 \\ -m_1 & -m_2 & m_1 & m_2 \\ -m_2 & -m_3 & m_2 & m_3 \end{bmatrix} \begin{matrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{matrix}$$

where,

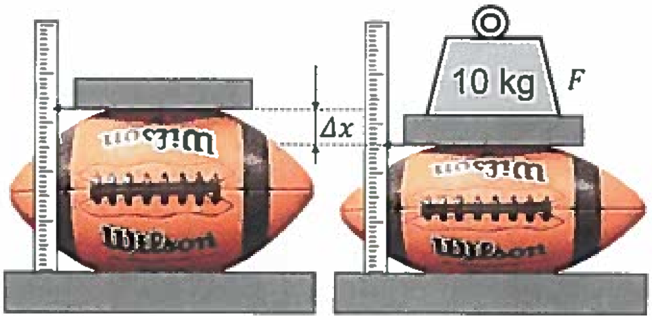
$m_1 = c^2 EA/L + s^2 EI/L^3$,

$m_2 = csEA/L - csEI/L^3$

$m_3 = s^2 EA/L + c^2 EI/L^3$,

and c and s are direction cosines of the element with respect to horizontal *(X)* and vertical *(Y)* axes.

Question 3: Numerical Analysis



The National Football League has strict guidelines on the allowable football pressure, and accordingly, the NFL decided to develop a simple test aparatus that can be used quickly and conveniently on the field to determine whether a ball is properly inflated. To avoid affecting the inflation pressure during the measurement, the choice is made not to insert a pressure gauge but instead to measure the stress-strain characteristic of the ball.

The proposed approach is the following. Place the ball in the aparatus between two fixtures, and measure the distance between the fixtures. Add a 10 kg weight and measure the distance again. Repeat for 20 kg, and 30 kg weights. The process is illustrated conceptually above.

By measuring the deformation, Δx, for different forces, F, an accurate assesment of the inflation pressure can be obtained. Wether the pressure is within the specified range is determined based on the parameters, k_1 and k_2 in the equation: $F = k_1 \Delta x + k_2(\Delta x)^2$. This equation indicates that the ball acts like a spring that stiffens as the deformation increases.

Your task is to come up with a numerical approach to determine the values for k_1 and k_2, given the measurements for Δx. For instance, you may have obtained the measurements: $\Delta x = 0.5$ cm for $F = 10$ kg, $\Delta x = 0.9$ cm for $F = 20$ kg, and $\Delta x = 1.3$ cm for $F = 30$ kg.

3.1) From the perspective of numerical methods, what type of problem is this?

3.2) Describe the equations or algorithm you would use to determine k_1 and k_2. Note: it is not necessary to perform the actual computations; to save some time, you can simply describe the computations symbolically. Describing the algorithm using Matlab code would also be an acceptable answer.

3.3) Did you make any assumptions in your approach? If so, what are they? Are they acceptable/justifiable? (Note: this is the most impart part of the entire question — explain it carefully).